Mathematical Analysis of Lateral Earth Pressure Distribution on Rigid Retaining Walls

[*]M. Ghazavi, M. MoshfeghYeganeh:
Faculty of Civil Engineering, K. N. T. University of Technology

Received: 8 Aprill 2011 Revised 4 Nov 2011

Abstract

The shape of slip surface of the wedge creating lateral thrust on rigid retaining walls plays an important role in the magnitude, distribution, and height of point of application of lateral thrust. Considering the shape of slip surface as linear, circular, logarithmic spiral, or a combination of them has been used in the literature. In the Coulomb lateral earth pressure method, a linear distribution of soil pressure on retaining walls is tentatively assumed and thus the point of application of total thrust is placed at one third of the wall height from the wall bottom. However, some experimental studies have revealed non-linear distribution of lateral earth pressures and that the point of application of resultant thrust is placed upper than one third of the wall height. In the present study, a plasticity equation is used to determine the reaction of the stable soil on cohesionless backfill supported by a retaining wall using an empirical equation derived from experiments performed in the field by others. A new analytical solution for determining the total resultant thrust on the wall is

[Downloaded from ndea10.khu.ac.ir on 2026-11-02]

[DOR: 20.1001.1.22286837.1391.6.2.2.6]

introduced and the distribution of pressures and the point of application of total thrust are computed. The results have been compared with some analytical methods, experimental data, and also with available data reported from field, demonstrating the accuracy and capability of the developed method. The results show that the distribution of the active lateral earth pressure is nonlinear and the point of application of total thrust is located about 0.42H from the wall bottom (H=wall height). In addition, the application point of total thrust is nonlinear function of soil-soil, wall-backfill soil friction angels and the height of the wall.

KeyWords: Retaining wall, Kotter equation, Active condition, Limit equilibrium method, Distribution of active earth pressure, Point of application of total thrust.

*Corresponding author ghazavi_ma@kntu.ac.ir

Introduction

The problem of retaining soil is the oldest subject in geotechnical engineering. Traditionally, civil engineers calculate the active earth pressure on the retaining walls using limit equilibrium methods the so called Coulomb's or Rankine's method. Both methods assume that the distribution of active earth pressure exerted against the wall is linear.

The prediction of maximum shear force and bending moment are important tasks in the design of retaining walls. These two are primarily functions of the earth pressure distribution. Also overturning moment will not be calculated accurately if the point of application of

[Downloaded from ndea10.khu.ac.ir on 2026-11-02]

[DOR: 20.1001.1.22286837.1391.6.2.2.6]

the lateral active force is determined inaccurately. Some experiments performed on retaining walls show the shape of the slip surface and the distribution of active earth pressure are nonlinear and also depend on the yielding mode of the wall. Additionally, experience in the construction of retaining walls often indicates that the quasihydrostatic pressure distribution does not always hold and the location of the resultant pressure on a retaining wall is higher than one third of the wall height (e.g., Terzaghi, 1934, Filroze, 1967, Matsuo, et al., 1978, Fang and Ishibashi, 1986). According to the general wedge theory (Terzaghi, 1941), if sufficient yielding of the wall is provided, the shape of the slip surface in the backfill depends not only on the soil-wall friction angle but also on the wall yielding mode. Terzaghi (1943) also believed that the slip surface developed in the backfill soil is planar if the wall back is smooth ($\delta = 0$ where δ is wall-backfill friction angle). For a rough wall, ($\delta \neq 0$), the slip surface becomes curved, irrespective of the wall yielding mode and if the wall yields by sliding, the slip surface will have a complex shape.

Tsagareli (1965) investigated the lateral pressure of wall backfill made of loose medium sand on large retaining wall model with a vertical back face and horizontal backfill surface and determined the magnitude, distribution and height of point of application of active earth pressure. According to this research, the slip prism has a curvilinear boundary. Also the normal earth pressure obtained in these experiments is close to that was calculated by Coulomb's formula.

[Downloaded from ndea10.khu.ac.ir on 2026-11-02]

[DOR: 20.1001.1.22286837.1391.6.2.2.6]

Based on his experiments, Tsagareli (1967) developed a mathematical solution for computing the distribution of lateral earth pressures in static condition by utilization the experimentally computed equations for the failure surface in the granular backfill. Base on this analytical method the distribution of the horizontal earth pressure on the retaining wall is nonlinear but the height of point of application of the total thrust is estimated about 0.37 H from the wall bottom.

Kobakhidze (1977) simplified the curved distribution of earth pressure with straight lines to obtain a center of pressure. This study shows that the location of the point of application of total thrust is located at about 0.396H-0.435H from the wall bottom with 0.95 probabilities. This may be reasonable for practice. On the basis of research work and a number of experiments, Kobakhidze (1977) recommends using 0.42H for point of application point of the total thrust. Many researchers further developed the limit equilibrium method. Wang (2000) developed mathematical solutions for computing the distribution of lateral earth pressures conditions. However, solutions are crude and need further improvement.

In the recent years, limit equilibrium approach has used to investigate lateral active force using horizontal slice method (Shekarian and Ghanbari, 2008; Reddy et al., 2008; Ahmadabadi and Ghanbari, 2009).This study develops an analytical solution for determination of magnitude, distribution and point of application of total thrust on rigid retaining walls. For this purpose, a nonlinear

[Downloaded from ndea10.khu.ac.ir on 2026-11-02]

[DOR: 20.1001.1.22286837.1391.6.2.2.6]

failure surface is assumed at soil-soil surface based on observation of Tsagareli (1965) in his experiments. The effect of soil properties and wall height on the lateral earth pressure distribution and height of application point of total thrust are highlighted.

Definition of shape of slip surface

As mentioned above, Tsagareli (1965) performed a series of large retaining wall model with a vertical back face and horizontal cohesionless backfill in active condition. He observed that the shape of slip surface in active condition can be well approximated with an exponential function represented by Eq. (1):

$$y = (0.5 + 3.6\varphi)^{x} - 1 \tag{1}$$

where φ is the soil internal friction angle, and x and y are coordinate of slip surface in plane

Determination of pressure on the failure surface

There are some assumptions required to determine the seismic active earth pressure. As shown in Fig.1, a fixed base vertical rigid retaining wall ($\xi = 90^{\circ}$) with H height is considered. The backfill is assumed to be homogenous, and dry cohesionless and with horizontal backfill surface ($\beta = 0$). Having the above assumptions and considering the shape for the backfill failure surface defined by Eq. (1), Kotter's equation based on the plasticity theory is used to compute the reaction of the stable soil on the failed wedge. This equation in the active condition is expressed as:

[Downloaded from ndea10.khu.ac.ir on 2026-11-02]

[DOR: 20.1001.1.22286837.1391.6.2.2.6]

$$\frac{dp}{ds} - 2p.\tan\varphi\frac{d\alpha}{ds}\gamma\sin(\alpha-\varphi) \quad (2)$$

where γ is the soil unit weight, α is the angle between the horizontal direction and tangential line at a given point on the failure surface, s is the arc starting from point A (Fig.2). Fig.1 shows pressure applied on failure surface for cohesionless soil in active and passive conditions.

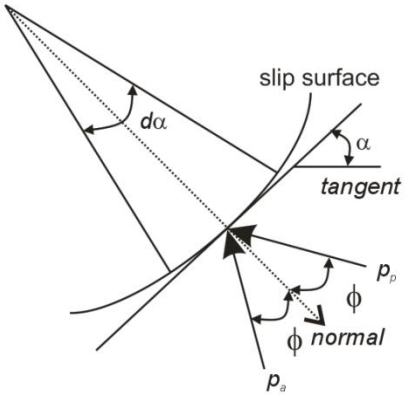


Fig.1. Pressure applied on failure surface for cohesionless soil in active and passive conditions

The total length of the arc from A to a given point on the failure is determined using:

$$S = \int ds \quad (3)$$

Where

$$ds = \sqrt{(dx)^2 + (dy)^2} = \sqrt{1 + (dy)^2} \quad (4)$$

Differentiation of Eq. (1) gives:

$$dy = (0.5 + 3.6\varphi)^x \log(0.5 + 3.6\varphi)^x \quad (5)$$

According to the Fig.1, α can be obtained from:

$$\tan\alpha = (0.5 + 3.6\varphi)^x \log(0.5 + 3.6\varphi) \quad (6)$$

Eq. (6) may be converted to:

$$\alpha = \tan^{-1}[(0.5 + 3.6\varphi)^x \log(0.5 + 3.6\varphi)] \quad (7)$$

Differentiation of Eq. (7) results in:

$$d\alpha = \frac{(0.5 + 3.6\varphi)^x \log(0.5 + 3.6\varphi)^2}{1 + (0.5 + 3.6\varphi)^{2x} \log(0.5 + 3.6\varphi)^2} dx \quad (8)$$

[Downloaded from ndea10.khu.ac.ir on 2026-11-02]

[DOR: 20.1001.1.22286837.1391.6.2.2.6]

As observed in Fig.2, assuming zero pressure at point A as a boundary condition and substituting Eqs. (4), (7), and (8) into Eq. (2), the differential equation of pressure is solved to give the distribution of reaction of the stable soil on the failed wedge, p, along AB surface.

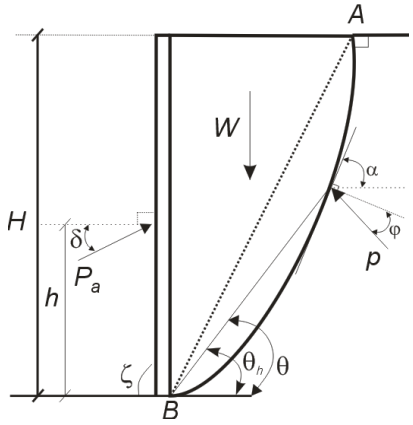


Fig.2. Reaction pressure on slip surface behind the retaining walls

Due to complicated shape of the slip surface and exponential function representing this surface, the differential equation of pressure should be solved using numerical integration methods (e.g. Explicit Runge-Kutta method).

Distribution of earth pressure on retaining wall

To determine the distribution of lateral earth pressure on a retaining wall, Fig.3a is considered in which a horizontal element of the backfill soil at a given depth y from the wall top is shown. Fig.3b illustrates the forces and stresses on the soil element. The mathematical solution is developed for the wall movement mode of translation by considering that the wedge slides as a whole. Based on the above

[Downloaded from ndea10.khu.ac.ir on 2026-11-02]

[DOR: 20.1001.1.22286837.1391.6.2.2.6]

assumption both shearing forces in the top and bottom of elements (inter slice forces) can be neglected. As shown in Fig.3b, the shearing forces on the top and bottom of the element are neglected.

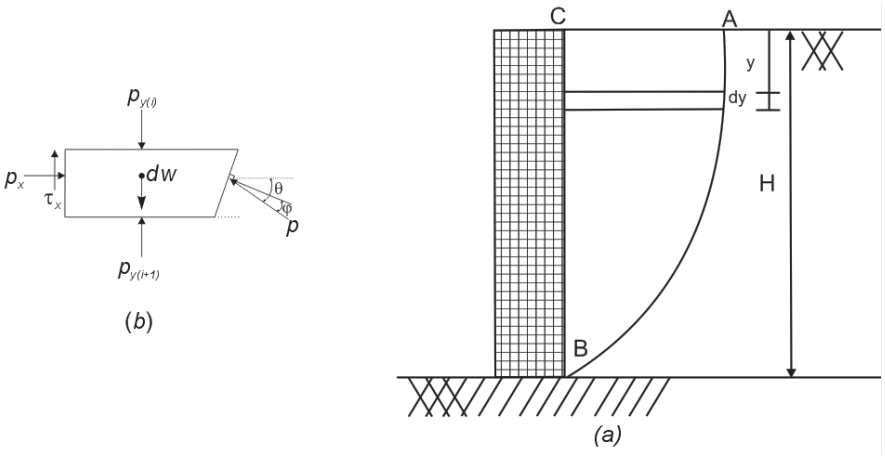


Fig.3. Retaining wall and backfill soil element for analysis of horizontal earth pressure distribution on wall

According to Fig.3b, the equilibrium of the horizontal forces on the backfill soil element results in:

$$p_x \times dy - p \times \cos(\frac{\pi}{2} - \alpha + \varphi) \times \frac{dy}{\sin\alpha} = 0 \tag{9}$$

which can be re-written as:

$$p_x = p \times \frac{\sin(\alpha - \varphi)}{\sin\alpha} \tag{10}$$

Fig.4 shows the distribution of the horizontal earth pressure on the retaining wall obtained from presented method and Rankine method. As seen, the distribution of horizontal earth pressure is nonlinear and its maximum does not occur at the toe of the wall. This distribution has similar shape as that obtained by Fang (1986) and also Tsagareli (1965).

[Downloaded from ndea10.khu.ac.ir on 2026-11-02]

[DOR: 20.1001.1.22286837.1391.6.2.2.6]

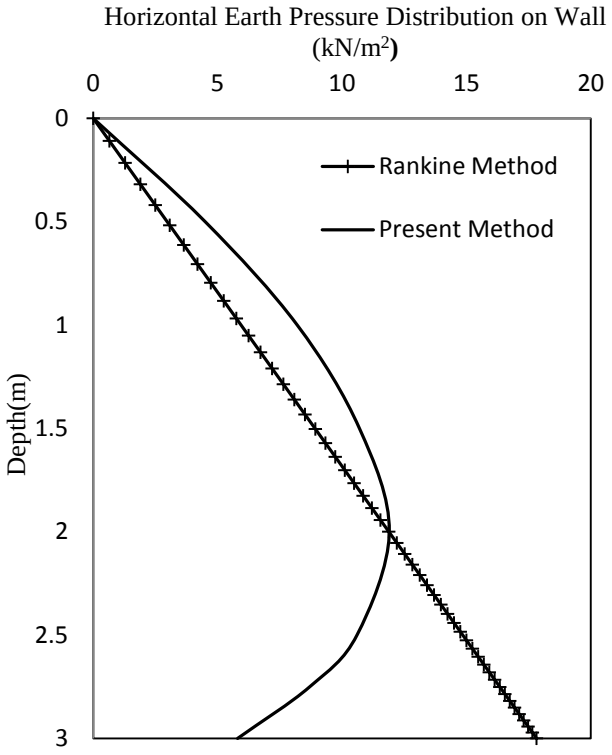


Fig.4. Distribution of horizontal earth pressure on retaining wall for $\varphi = 30^{\circ}, \delta = 0, H = 3m, \beta = 0, \xi = 90^{\circ},$ $\gamma = 18\ \frac{kN}{m^3}$

Determination of total thrust

Total thrust can be determined by integrating lateral earth pressures on the wall. Fig.5 shows the variation of total active thrust on the wall with friction angle of backfill. As observed, for friction angles greater than 33^{o}, Coulomb method estimates greater total thrust than computed using preset method by utilization of the experimental slip surface. As seen, with increasing the backfill friction angle, the total lateral thrust decreases.

[Downloaded from ndea10.khu.ac.ir on 2026-11-02]

[DOR: 20.1001.1.22286837.1391.6.2.2.6]

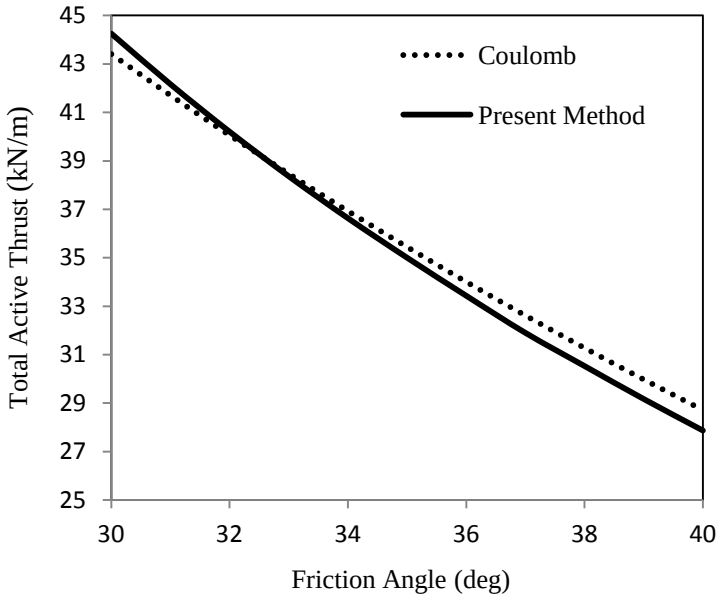


Fig.5. Variation of total active thrust with friction angle of backfill for $\delta = \frac{\varphi}{2}, H = 4\,m\,, \beta = 0, \xi = 90^{\circ}, \gamma = 18\,\frac{kN}{m^3}$

The point of application of the total thrust can be calculated by considering the moment equilibrium about the wall toe. Due to nonlinear distribution of earth pressure and considering that the maximum earth pressure does not occur at the toe of the wall, the point of application of total thrust places at an upper point than one-third of the wall height from the wall bottom.

In Table 1, the height of point of application of total active thrust obtained from the present method is compared with experimental data reported by Tsagareli (1965) from full-scale experiments on translating walls. These experimental results confirm a curvilinear distribution for lateral earth pressures rather than the classical triangular distribution.

[Downloaded from ndea10.khu.ac.ir on 2026-11-02]

[DOR: 20.1001.1.22286837.1391.6.2.2.6]

In order to demonstrate the accuracy and capability of the presented solution, the heights of the application point of the lateral active thrust computed from other analytical methods (Tsagareli, 1967, Coulomb method) have been compared with those obtained from the developed solution in Table 1. As seen, the heights of the point of application of the total active thrust obtained from the present method have reasonable agreement with experimental data reported by Tsagareli (1965) and different from the values obtained from conventional methods based on the Rankine and Coulomb solutions.

Table.1. Comparison of points of application of total active thrusts from several methods for

$$\varphi = 37^{\circ}, \delta = 0, \beta = 0, \xi = 90^{\circ}, \gamma = 18\frac{kN}{m^3}$$

Distance of point of application of total thrust from wall bottom	Coloumb	0.33H	0.33H	0.33H	0.33H	0.33H
	Tsagareli (experimental, 1965)	0.43H	0.42H	0.43H	0.41H	0.41H
	Tsagareli (analytical, 1967)	0.36H	0.37H	0.36H	0.36H	0.36H
	Present study	0.42H	0.428H	0.429H	0.427H	0.425H

It is noted that the location values of the active forces determined from the developed solution lie in the range summarized by Kobakhidze (1977).

Table 2 compares the height of the point of application of the lateral active force predicted by the proposed method with the measurements of Fang and Ishibashi (1987). As seen, the height of points of application of total thrust calculated from the proposed method is comparable with measured values.

[Downloaded from ndea10.khu.ac.ir on 2026-11-02]

[DOR: 20.1001.1.22286837.1391.6.2.2.6]

Table2. Comparison of points of application of total active thrusts for various friction angles

$\beta = 0, \xi = 90^{\circ}, \gamma = 18 \frac{kN}{m^3}$

Distance of point of application of total thrust from wall bottom	Method	Friction Angel					
		32^o	34^o	36^o	38^o	40^o	42^o
	Fang and Ishibashi (1967)	0.4H	0.41H	0.42H	0.428 H	0.433 H	0.44H
	Present study	0.405 H	0.409 H	0.414 H	0.42H	0.425 H	0.435 H

Fig.6. shows the variation of h_r with the backfill friction angle determined from the presented method where $h_r = \frac{h}{H}$ and h is distance between the total thrust from the wall bottom. As seen, the height of the point of application of the total thrust increase nonlinearly with □□and this height is a function of soil properties

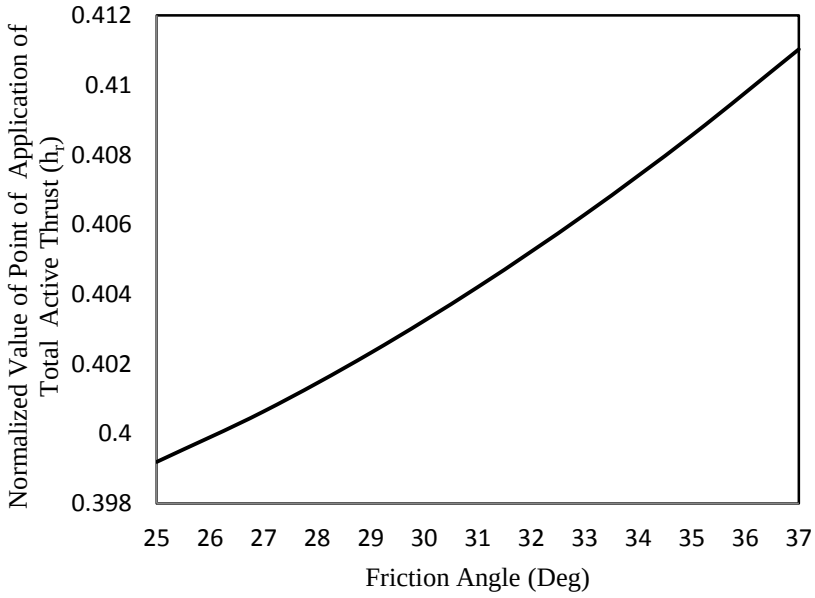


Fig.6. Variation of height ratio (h_r) with friction angle determined from present analysis for

$H = 10\ m,\ \gamma = 18\ \frac{kN}{m^3}$

[Downloaded from ndea10.khu.ac.ir on 2026-11-02]

[DOR: 20.1001.1.22286837.1391.6.2.2.6]

Fig.7 shows the variation of h_r with wall height determined from the presented method. As seen, the height of the point of application of the total thrust varies nonlinearly with H and it is a function of the wall height. It should be noted that some methods neglect the effect of wall height on the determination of application point of the total thrust, for example, Paik (2003).

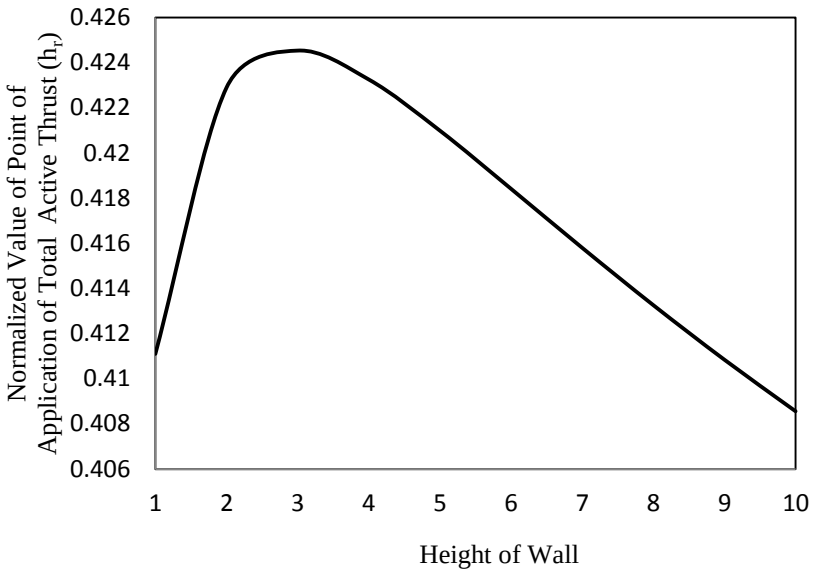


Fig.7. Variation of height ratio of point of application of active thrust with height of the wall determined from present analysis for $\varphi = 35^{\circ}$, $H = 10\ m,\ \gamma = 18\ \frac{kN}{m^3}$

Conclusions

An analytical solution was presented in this paper in which a curved failure surface is assumed to occur in granular backfill behind a rigid retaining wall. Using the limit equilibrium approach combined with Kotter's equation, an analytical solution for determination of

[Downloaded from ndea10.khu.ac.ir on 2026-11-02]

[DOR: 20.1001.1.22286837.1391.6.2.2.6]

total active thrust, its application point, and the lateral earth pressure distribution on the wall was developed. The analysis data were compared with available data reported from experiments, resulting in satisfactory agreement. The results show that the distribution of the horizontal earth pressure on the retaining wall is nonlinear and the height of point of application of the total thrust is estimated about 0.42 H from the wall bottom. In addition, it was found that the height of the point of application of the total thrust varies nonlinearly with the backfill soil-soil, soil-wall friction angle, and the wall height.

References

1. Ahmadabadi, M., Ghanbari, A., "New procedure for active earth pressure calculation in retaining walls with reinforced cohesive-frictional backfill", Geotextiles and Geomembranes, No. 27 (2009) 456-463.
2. Das, B. M., "Theoretical foundation engineering", Ross Publication Edition (2007).
3. Fang, Y and Ishibashi, I. "Static earth pressures with various wall movements", Journal of Geotechnical Engineering. ASCE, Vol. 112, No. 3 (1986) 317-333.
4. Fil'roze, R. M., "Experimental investigations of the earth pressure on a retaining wall", Discussions on applied soil mechanics No. 3 (1967) 47-50.
5. Kobakhidze, A. F. "Form of the diagram of backfill pressure on a retaining wall", Soil Mechanics and Foundation Engineering, Vol.

[Downloaded from ndea10.khu.ac.ir on 2026-11-02]

[DOR: 20.1001.1.22286837.1391.6.2.2.6]

14, No. 1 (1977) 68-73 (translation from Osnovoniya Fundamenty i Mekhanika Gruntov ,No. 1, Jan.-Feb., (1977) 37-40).

6. Matsuo M., Kenmochi S, Yagi H., "Experimental study on earth pressure of retaining wall by field test", Soil Mechanics and Foundation Eng. 18(3) (1978) 27-41.
7. Paik, K., Salgado, R., "Estimation of active earth pressure against rigid retaining walls considering arching effects", Geotechnique, No. 53(7) (2003) 643-653.
8. Reddy, G. V. N., Madhav, M. R., Reddy, E. S., "Pseudo-static seismic analysis of reinforced soil wall: effect of oblique displacement", Geotextiles and Geomembranes 26 (5) (2008) 393-403.
10. Richards, R. Jr., "Principles of solid mechanics. SUNY, Buffalo", New York, USA Publication (2001).
11. Sherif, M. A., Ishibashi, I., Lee, C. D., "Earth pressure against rigid retaining walls", Journal of Geotechnical Engineering, ASCE, Vol. 108 (GT5) (1982) 679-693
12. Sherif, M. A., Ishibashi, I., Lee,C. D., "Earth pressure against rigid retaining walls", Journal of Geotechnical Engineering, ASCE, Vol. 108(GT5) (1982) 679-693.
13. Sherif, M. A. and Fang, Y. S., "K_a and K_o behind rotating and non-yielding walls", Journal of Geotechnical Engineering, ASCE, Vol. 110, No. 1(1984) 41-56.
14. Shekarian, S., Ghanbari, A., Farhadi, A. "New seismic parameters in the analysis of retaining walls with reinforced backfill", Geotextiles and Geomembranes 26 (2008) 350-356.

[Downloaded from ndea10.khu.ac.ir on 2026-11-02]

[DOR: 20.1001.1.22286837.1391.6.2.2.6]

15. Terzaghi, K. "Large retaining wall tests, Eng News Record", Vol. 112 (1934) 136-140.
16. Terzaghi, K., "General wedge theory of earth pressure", ASCE, Vol. 106, NO. 2099 (1941) 68-97, 15 FIG, 2 TAB.
17. Terzaghi, K., "Theoretical soil mechanics", John Wiley and Sons, Inc., New York (1943).
18. Tsagareli, Z. V., "Experimental investigation of the pressure of a loose medium on retaining walls with a vertical back face and horizontal backfill surface", J. Soil Mech. Found. Eng, ASCE 91, No. 4 (1965)197-200.
19. Tsagareli, Z. V. "Method of determining the pressure of loose materials on retaining walls by utilization of experimentally determined equations for the surface of creep", Translated from Gidro tekhnicheskoe Stroitel'stvo, No. 6 (1967) 44-48.
20. Wang, Y.Z., "Distribution of earth pressure on a retaining wall", Ge´otechnique, Vol. 50, No. 1 (2000) 83-88.

[Downloaded from ndea10.khu.ac.ir on 2026-11-02]

[DOR: 20.1001.1.22286837.1391.6.2.2.6]