Effect of Matching Period-Interval Variation on Strong Ground Motion Scaling

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Abstract

Time history analyses as crucial means in many earthquake engineering applications are highly dependent to characteristics of the seismic excitation record so that the resulting responses may vary from case to case. Strong ground motion scaling is a known codified solution to reduce such a dependency and increase reliability of time history analyses. The well-known code practice may result in highly non-economic designs due to considerable error in the spectra scaled to match the target code spectrum. This problem is formulated here in an optimization framework with the scaling coefficients as the design variables. Harmony search as a recent meta-heuristic algorithm is utilized to solve the problem and is applied to the treated examples. Using a variety of target period ranges the scaling error is evaluated and studied after more unified via optimization. The effect of base structural period and interval variation on the scaling error is then studied in addition to considerable error decrease with respect to traditional code-based procedure. The results also show dependency of spectral matching error to the period-interval elongation/variation,

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the base-structural period and more error sensitivity for narrow-band resonance with the filtered records on softer soil types.

KeyWords: Strong Ground Motion, Optimal Spectral Scaling, Matching Period Interval, Harmony Search

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Introduction

Seismic excitation for structural and geotechnical applications can be expressed in the form of recorded earthquake accelerograms. In this regard some sources for providing time-history records can be distinguished: recorded experienced earthquakes, statistically simulated accelerograms or model-based artificial records [1-3]. The first group is preferred in many practical cases because it conserves the frequency content and other characteristics of the real-world records in spite of artificial records. However, for a specific site the forthcoming earthquake cannot be deterministically predicted yet, even from the previous earthquakes.

Therefore, statistical analyses are performed to derive mean plus one standard deviation spectra. They are further smoothed and classified considering simplified interaction effects of various soil types in the design codes and amended as the mere legal source of seismic loading in terms of design spectra [3, 4]. Such design spectra are only sufficient for modal analyses of linear systems [5]. However,

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many practical applications needs time history analyses of soil or structural system as numerical source of seismic excitation [6-8]. Thus, a procedure to make a recorded time history accelerogram compatible with the code-based design spectrum for the site of construction with specific soil type is needed [9-11]. It is called the scaling procedure already offered by many codes of practice which is being reviewed in detail in the next section.

According to such a procedure, the averaged spectrum extracted from a number of time-history records is scaled to match the design spectrum with in a prescribed period range. The present study first formulates it as an optimization problem to minimize such a spectral compatibility error and then concerns effect of the target period range on it. The recently developed Harmony Search algorithm is thus specialized for this optimization problem [12, 13]. A variety of period range classes are then considered for further parametric study in order to derive the error curve for each distinct class and soil-type. Final concluding remarks are then driven discussing and comparing the achieved results.

The Scaling Procedure

According to the current design codes [4-5], seismic excitation is legally introduced in terms of a few design spectra rather than time-history records. It is due to the fact that no special time-history can be exactly predicted for all sites of construction, but design spectra are more reliable when generated based on extensive statistical operations

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on several previous earthquakes over the world or country. Such well-known seismic design codes have thus offered a procedure to modify available set of accelerograms for a specific site of construction so that their corresponding spectra are compatible with the legal design code spectra.

The procedure is called ground motion scaling and is given via following steps according to the Iranian standard 2800-84 [4]. According to it, N pairs of horizontal earthquake components are normalized to their Peak Ground Acceleration, PGA. Then response spectra of each pair is calculated and combined to generate a Square Root of Sum of Square, SRSS spectrum. Average of these N SRSS spectra are then compared with 1.4 times the standard spectrum within a, Matching Period Interval, MPI and scaled so that not fall below such target in the employed MPI. The resulting scale factors are then used to amplify the records before being employed as time history analysis input.

The aforementioned single-value scaling procedure is preferred to other simulation methods because it only amplifies the accelerogram magnitude preserving its frequency content and non-stationary characteristics of the initial time history. As can be realized in such scaling procedure, the resulted scale is dependent to the employed MPI; given $[0.2T_{Str}, 1.5T_{Str}]$ due to Iran seismic design standard-2800. The base period T_{Str} denotes natural period of the structure and is determined computationally considering empirical design code

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relations [4]. In addition, the design code allows taking simple average as mean spectrum that usually results in non-economic over-conservative values for the scaled spectrum far larger than the target. Much better weighted mean coefficients can be search to reduce such a compatibility error via optimization as dealt in the next section of the present article.

Optimized Scaling Using Harmony Search

As mentioned above, the compatibility of average scaled spectrum with the design target can be maximized by optimization. In this regard, the problem is formulated as below when the coefficients to compute such an optimal weighted average, are denoted by X; i.e., vector of optimal design variables:

$$\begin{aligned} &Minimize \quad MatchingError(x_1,...,x_N) = 100\sum_{T_1}^{T_2}\left|\frac{\beta.\overline{SA}(T) - SA_{T\arg et}(T)}{SA_{T\arg et}(T)}\right| \\ &S.t. \qquad\qquad 0 < x_i \le 1 \end{aligned} \tag{1}$$

$$\overline{SA}(T) = \frac{\sum_{i=1}^{N} x_i SA_i(T)}{N} \tag{2}$$

$$\beta = \max_T\{\frac{.SA_{T\arg et}(T)}{\overline{SA}(T)}\} \tag{3}$$

Whereas $SA_{T\arg et}(T)$ is spectrum given by the design code at any period; T. The spectral value $SA_i(T)$ stands for the spectral SRSS acceleration for each earthquake and $\overline{SA}(T)$ denotes the corresponding weighted average spectrum using scale factors: x_i . The coefficient β is

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used to insure $\overline{SA}(T)$ does not fall below $SA_{T\arg et}(T)$ among periods in the interval T_1 to T_2 .

Once the optimization problem is defined, an algorithm should be employed to search for its optimal vector of continuous scaling design variables x_i in range (0, 1].

Harmony Search, HS, as a recent optimization algorithm is inspired by the method a musician makes new notes considering its Harmony Memory, HM [12]. As this algorithm is best suited for continuous search spaces, it is utilized to optimize the design variables of the scaling problem in the present work. The algorithm is implemented via the following steps:

1. Initialize the harmony memory: pick k random vectors; X^1, X^2, X^3, ... , X^k

2. Make a new vector X'. For each component x'_i: X_i'=X_i rand()

with probability p_{hmcr} pick the component from memory,

with probability $1 - p_{hmcr}$ pick a new random value in the allowed range.

3. Pitch adjustment: For each component x'_i:

with probability p_{par} change x'_i by a small amount, $\pm b_w$.rand.

with probability $1 - p_{par}$ do nothing.

4. If X ' is better than the worst X_i in the memory, then replace X_i by X '.

5. Repeat from step 2 until a maximum number of iterations has been reached.

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According to the above algorithm, required control parameters are distinguished as:

k, the size of the memory.

p_{hmcr}, the rate of choosing from memory, HM.

p_{par}, the 'pitch adjustment rate'.

b_w, the 'bandwidth' or the amount of change for pitch adjustments.

It is possible to vary the parameters as the search progresses; this gives an effect similar to simulated annealing. In the improved harmony search, p_{par} is increased linearly, while b_w is decreased exponentially.

The parameter values used in this study are listed in table 1 and fitness function of each coefficient group X is identified based on the resulting spectral matching error. The fitness function is taken $Fitness(X) = -MatchingError(X)$ and is to be maximized to minimize the spectral matching error.

In order to make a comparison scaling result, a sample set of records are shown in Figures 1 and 2. As can be realized from Figure 1 the manual practice as code using similar weighting factors has led to the maximum compatibility error, while it is decreased by Genetic Algorithm, GA and more decreased by HS. GA parameters in this sample run are taken 90% for crossover and 5% for mutation probability thresholds where others are taken similar to HS parameters in Table 1.

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As maximizing the fitness corresponds to minimizing the spectral matching error, its trace vs. iterations of the search is demonstrated in Figure 2. It is worth mentioning that the fittest design vector in each iteration is saved and replaced with the least fit one in the next iteration. According to Figure 2 HS fitness history stands higher than sample GA continuing its progress. This shows comparable and even better performance for HS with respect to well-known GA for such parameters in the treated scaling problem.

Table1. Control parameters of the employed HS algorithm

Population Size (k)	P_{hmcr}	P_{par}	Bandwidth, bw	Number of Iterations
10	0.90	0.30	2	1000

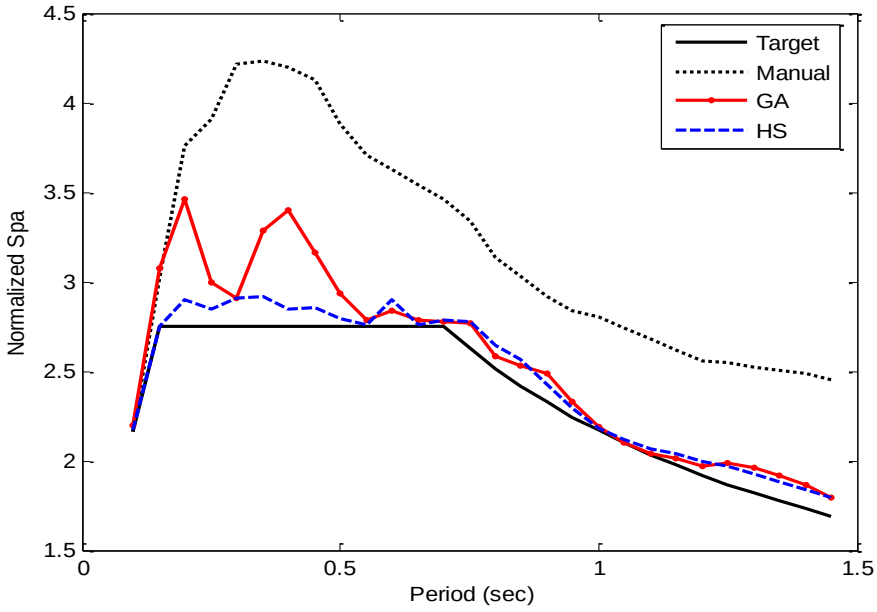


Figure1. Comparison of sample scaled mean spectra against the design target for soil-III

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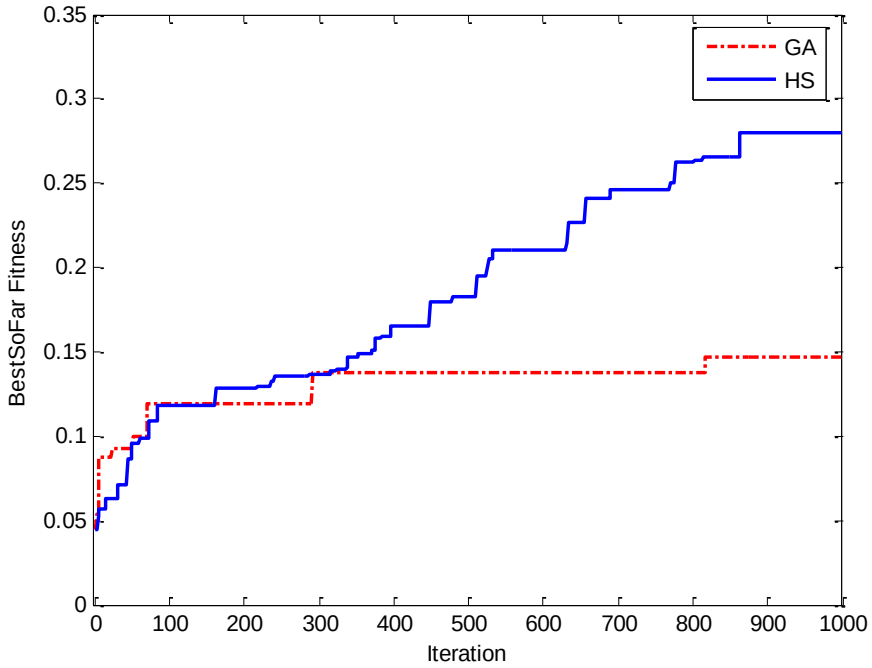


Figure2. Sample convergence curves obtained by HS and GA

Considered Classes of Matching Period Intervals

The consequent parametric study is concerned with the effect of various soil types, micro/macro earthquake characteristics, the base period shift and the way that corner periods of the spectral matching region are determined. Two main classes are distinguished in this regard:

a) Variable-length MPI

b) Constant-length MPI

The first class is itself subdivided into MPI of linear or non-linear structural behaviors. For the 1st case suppose building structure is linear and be approximated by a flexural equivalent beam. According

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to the solution of governing differential equation of motion for a uniform cantilever beam with fixed-free end conditions, its nth period, T_n, can be estimated as [14]:

$$T_n = \frac{1}{2n-1} T_1 \tag{4}$$

where T_1 is the fundamental period of the structure. This MPI class in the current research is thus taken a range of $\frac{1}{2n} T_1$ to T_1 covering all periods of the n-story building in a linear-model.

$$MPI_{VF} = [\frac{1}{2n} T_1, T_1] \tag{5}$$

It is worth noting that several building frames experience period elongation due to formation of plastic zones during earthquake loading; therefore another class of MPI regarding non-linear modes is also considered here as:

$$MPI_{NL} = [(1-0.8a)T_1, (1+0.5a)T_1] \tag{6}$$

in which the extension or dilatation of MPI is dominated by the elongation factor, a . Applying $a = 1$ is identical to use the recommended relation in UBC97 and Iranian Standard-2800 [4, 5]:

$$MPI_{Code} = [0.2T_1, 1.5T_1] \tag{7}$$

In both these classes, the more the building natural period is, the larger MPI is exerted to the spectrum-matched scaling procedure; so they address variable-length MPI's.

Fixing the natural period of base structure, leads to constant-length MPI. In this study, once a fixed period of $T_1 = 1.7s$ is considered as a representative for common building periods due to the Standard-2800

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regulations [4]. Applying $T_1 = 1.7s$ in Equation (7) results in a constant-length interval here-in-after called; MPI_{CM2800}.

Once again the natural period of a mid-height 7-story building by the 2800-standard imperial relations is used as T_1 in Equation (7) to construct another constant-length; MPI_{CMF}. Either MPI_{CM2800} or MPI_{CMF} is then shifted as T_1 varies to trace the resulted spectral matching error, provided that the MPI length is taken constant.

Employing the optimization tool, effects of the aforementioned types of MPI on the resulting error are studied in the next section.

Numerical Results

The Iranian Standard-2800 considers 4 soil types and 2 seismicity classes to generate the corresponding design spectra for each case. Soil types are classified based on the average shear wave velocity in the upper 30m ground depth, $\overline{V}_s$, such as given in Table 2. Regional differences are also distinguished as low-medium or high - very high seismicity classes in Tables 3 and 4. Design base PGA for the corresponding seismicity region are given in Table 3. The other code specific parameters do not affect the scaling factors and compatibility as they are calculated for the normalized ground motion. Once the aforementioned parameters are determined, the target design spectra can be then constructed according to standard-2800 as depicted in Figure 3.

The modified accelerogram set for the present parametric study are then provided from the PEER database [15]. Different record sets are

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extracted for different classes of soil-types, magnitudes/seismicity. Table 5 summarizes the criteria used in this study to provide each set of input earthquake records from PEER Database search-engine. As the Iranian standards 2800 spectra does not explicitly cover near field motion, only far field records are concerned here.

Figure 4 demonstrates variation of optimal fitness vs. the n-story steel moment-frame period, used in the MPI_{Code} for the low-medium seismicity. It is shown that scaling to the target spectrum of soil type 3 has caused less sensitivity than other soil types to period variation regarding the spectral matching error. Note that the less fitness is the more spectral error is resulted due to relation (1).

In addition, the spectral matching error has its highest values for the soil type 4 and is increasing with base period for relatively stiff soils 1 and 2. Inspecting the matter for high or very high seismicity declares considerably more sensitivity of error to the period for the soil type 4 (Figure 5).

Table2. Considered soil type classes in the present work

Soil group	1	2	3	4
$\overline{V}_s$ (m/s)	>750	375~750	175~375	< 175

Table3. Design base PGA's in different seismicity regions according to [4]

Seismicity	Low	Medium	High	Very high
$PGA\ \ (g)$	0.20	0.25	0.30	0.35

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Table4. Maximal amplification factors due to regional seismicity in the acceleration design spectra [4]

Seismicity	Low or Medium	High or Very high
B_{max}	2.5	2.75

Table5. Considered sets to provide ground motion from PEER database [15]

Record set	Earthquake magnitude (Richter)	Closest distance from Source (km)	$\overline{V}_s$ (m/s)	$PGA\ \ (g)$	Number of available records
1-1	6~9	>15	>750	0.01~0.25	50
1-2				0.25~0.35	7
2-1			375~750	0.01~0.25	50
2-1				0.25~0.35	7
3-1			175~375	0.01~0.25	50
3-1				0.25~0.35	14
4-1			< 175	0.01~0.25	33
4-1				0.25~0.35	1

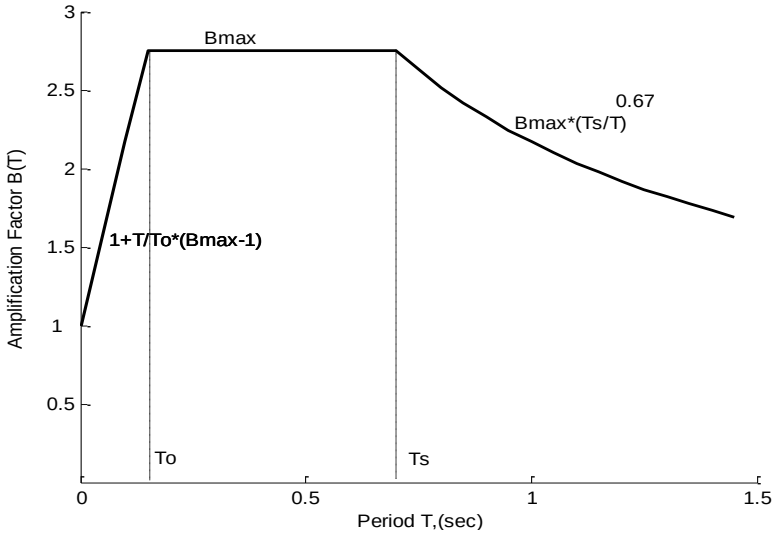


Figure3. Typical target design spectrum according to [4]

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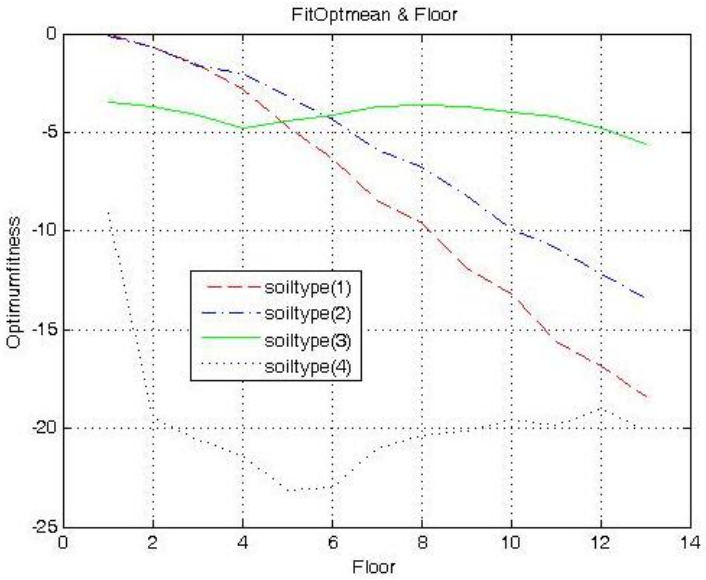


Figure4. Fitness variation vs. changes in number of building stories (floors) in variable length MPI_{Code} for different soils in low-medium seismicity

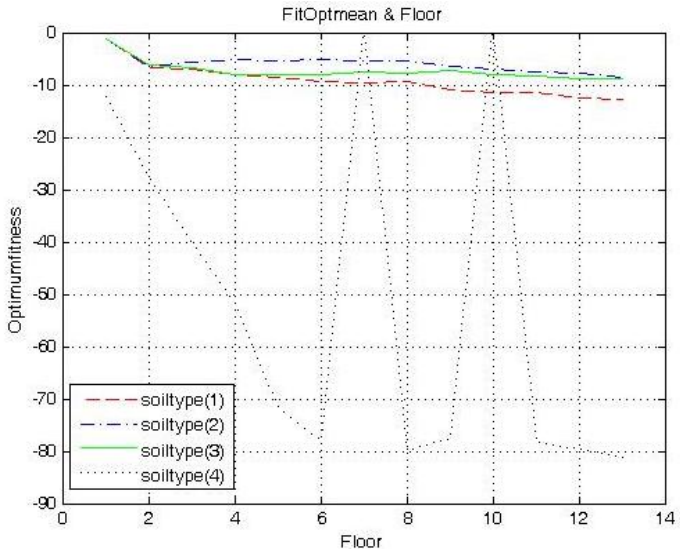
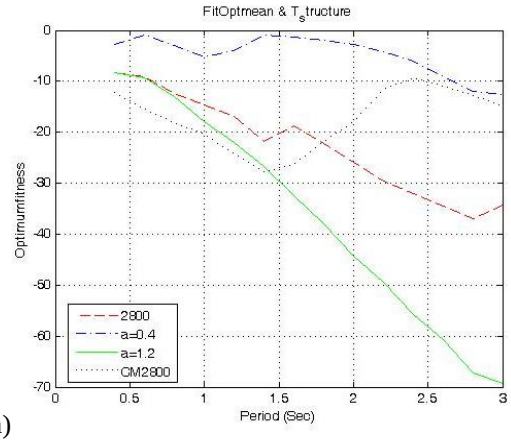


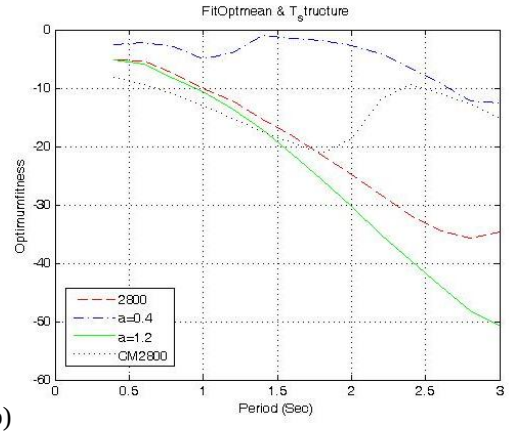
Figure5. Fitness variation vs. changes in number of building stories (floors) in variable length MPI_{Code} for different soils in high-very high seismicity

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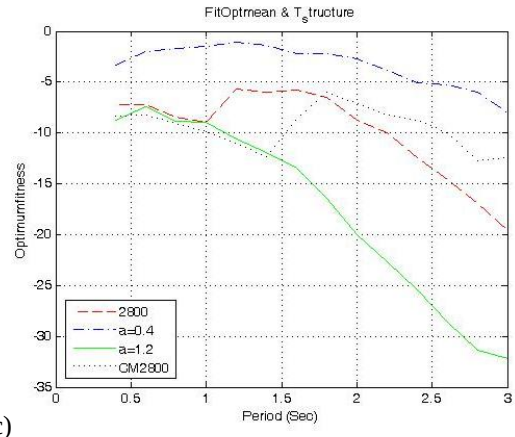
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(a)



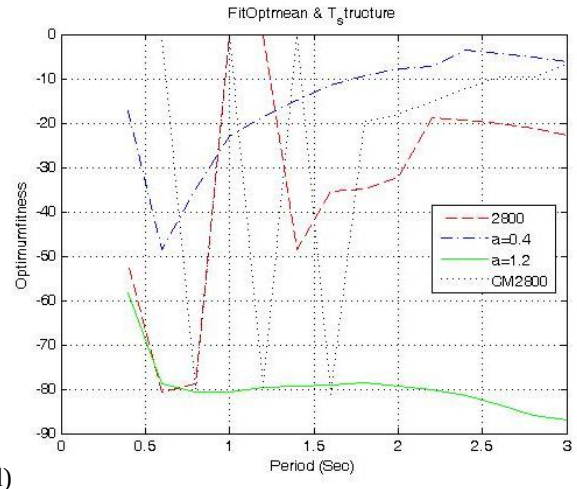
(b)



(c)

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(d)

Figure6. Fitness changes with variation of MPI_{NL} elongation factor, a, in high-very high seismic hazard area for different soil types (a) 1, (b) 2, (c) 3, (d) 4

In Figure 6 the effect of variation in period interval length is traced by changing the elongation factor, a, for MPI_{NL} with different values of 0.4, 1 and 1.2. Note that the value 1 for a results in the MPI of Standard-2800. According to Figure 6 the maximal spectral matching error is decreased when altering the soil type from 1 to 3 from almost 70% to 35%) but suddenly increased for the soil type 4. It can be notified that the more the elongation factor, a, is used the smoother curve is obtained in nearly all the cases. Using constant MPI, CM2800 has also resulted in more fluctuation of such curves in certain (middle) periods. Note that shorter matching interval acts like a smaller moving window which is more sensitive to the total variation of the scaled average spectrum with respect to the design target. Figures 7-9 reveal comparison of soil types for each of the employed a-factors. The trend

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of fitness (opposite of error) variation vs. base period is almost similar for soil types 1, 2 and 3 but different for type 4; especially for more elongated MPI's. Considering Table 5, such an unusual behavior of soil type-4 can be declared. It may be related to very few number of available records in soil 4 with respect to other types that affects smoothness of the resultant mean spectrum subject to be scaled and made compatible with the smooth target spectrum.

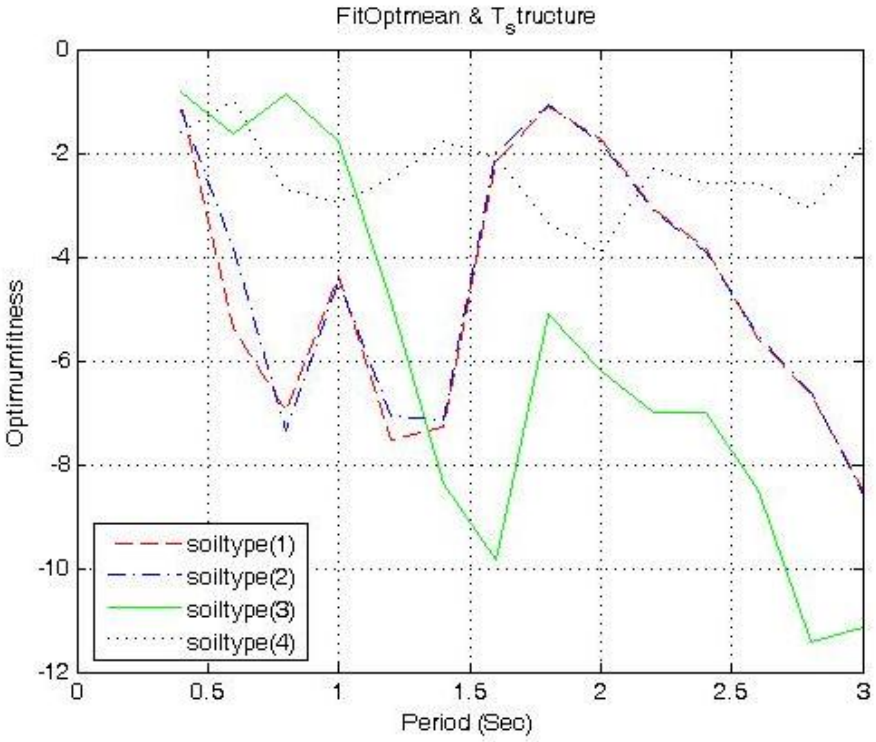


Figure7. Spectral compatibility of various soil types vs. MPI_{NL} in low-medium seismicity region for elongation factor of 0.4

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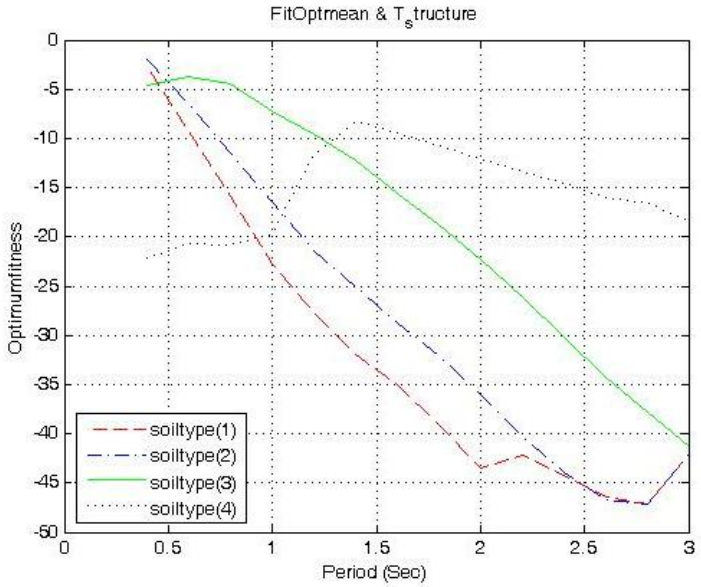


Figure8. Spectral compatibility of various soil types vs. MPI_{NL} in low-medium seismicity region for elongation factor of 1.0 (which coincides MPI_{Code} relation due to Standard-2800)

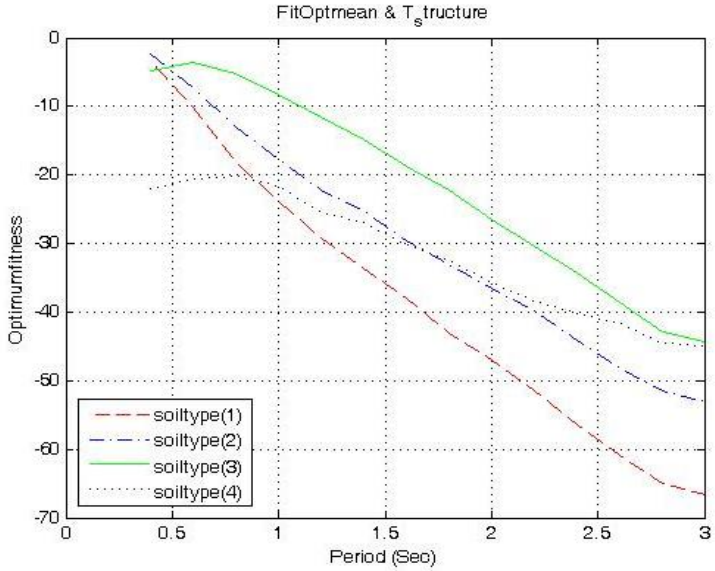


Figure9. Spectral compatibility of various soil types vs. MPI_{NL} in low-medium seismicity region for elongation factor of 1.2

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According to Figure 10, the trend of the variable-length MPI_{VF} is similar to the constant-length MPI_{CM2800} for linear and rigid systems with low natural period. However, the spectral error (opposite of fitness) is much lower in MPI_{VF} than MPI_{CM2800} for greater periods or taller building frames.

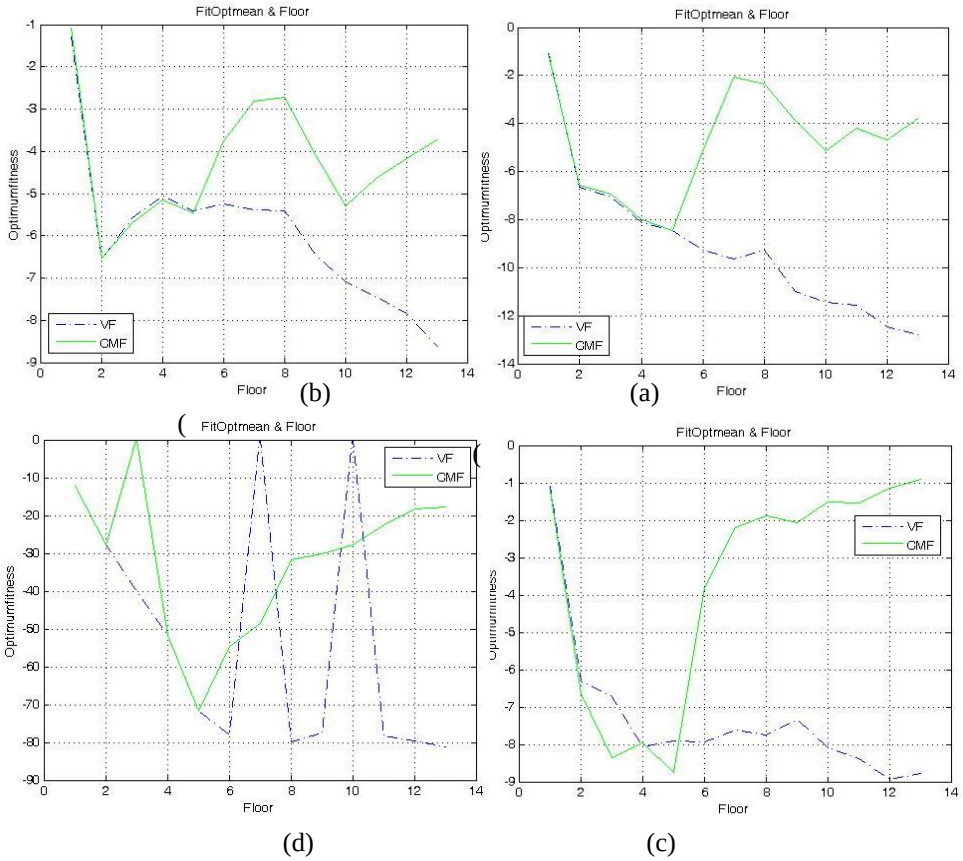


Figure10. Comparison of fitness variation between variable-length MPI_{VF} and constant-length MPI_{CM2800} in high seismicity for different soil types (a) 1, (b) 2, (c) 3, (d)4

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Summary and Conclusion

Strong ground motion scaling is used to match the real records of past earthquakes with the design code requirements without changing their frequency content. Such a spectral matching is performed within a prescribed period range. In order to minimize the spectral compatibility error, the problem is formulated with scaling factors as continuous design variables. Harmony Search as a recent meta-heuristic algorithm is then utilized for the proposed optimization problem and implied in each case to drive the optimal scaling factors and reveal the corresponding compatibility error for further parametric study.

According to the obtained results, using softer soils' spectra as the target for scaling generally causes more difference between maximum and minimum spectral compatibility error. Besides, variable-length intervals with MPI_{Code} when expanded by a-factor greater than 1 led to smoother fitness or error history among the entire period range.

The trend of fitness variation was similar comparing soil types 2 and 1; however, it showed higher numbers of picks in the former. Such a similarity was also observed for soil type 4 with respect to type 3 but with many more picks and fluctuation. More intense filtering of the source record's frequency content by the softest soil type 4 can be a reason for that, however, conclusion on this type of soil requires more records for an intense study.

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Variable length MPI's had smoother error curves than constant-length intervals in frequency domain considering the shape of target design code spectra and its amplified constant-acceleration region.

Additionally, the trend of fitness changes in low-medium seismic hazard area obtained smoother than that for the case of high/very high seismicity. In the other hand, multi-shock nature of greater magnitude earthquakes can result in more seismicity and thus be related to such a more sever variation of fitness due to their higher frequency content with many picks in the resulted spectra.

Sensitivity of fitness function to period variation was found the higher in the softer soil types. It seems that the increased spectral compatibility errors are concentrated around some certain periods amplified when filtered by soft soil fundamental periods.

Finally light of the obtained results in the current research, it is declared that the following factors can greatly affect the resulting optimal fitness or spectral matching error:

- Resonance between fundamental periods of the soil and the structure
- Variation of period-interval length
- Earthquake's multi-shock class and consequent regional seismicity
- Soil type classes and effects on filtering of seismic waves by them

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It was also shown that considerable error reduction is achieved by optimized scaling factors with respect to common code practice. Thus the presented algorithm can be recommended for optimal ground motion scaling; however, special attention should be paid when scaling to the target spectra of soft soils for high-rise buildings. Regarding optimization parametric study, near field records, source magnitude variation, statistical issues and more smooth mean spectra are of future scope of research.

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